

Quantitative Probe of Pairing Correlations in a Cold Fermionic Atom Gas

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(Dated: February 2, 2008)

A quantitative measure of the pairing correlations present in a cold gas of fermionic atoms can be obtained by studying the dependence of RF spectra on hyperfine state populations. This proposal follows from a sum rule that relates the total interaction energy of the gas to RF spectrum line positions. We argue that this indicator of pairing correlations provides information comparable to that available from the spin-susceptibility and NMR measurements common in condensed-matter systems.

PACS numbers: 03.75.Kk, 67.40.-w, 32.80.Pj

Introduction — The realization of degenerate atomic Fermi gases [1, 2, 3, 4, 5, 6, 7] has opened new opportunities for experimental discovery. The focus to date has mainly been on efforts to observe the condensation of atomic Cooper pairs to form a superfluid state similar to the BCS state of electrons in a superconductor [8]. Strategies for achieving observable pairing effects have so far hinged on the occurrence of strong attractive atom-atom interactions near a Feshbach resonance [9, 10, 11], in which a molecular bound state of one atom-atom scattering channel is close to the two-atom continuum threshold of another. The proximity of a Feshbach resonance can be adjusted by tuning a magnetic bias field, drastically altering the scattering behavior of atoms and allowing the s -wave scattering length to be varied over values corresponding to effective interactions that are weak or strong, and repulsive or attractive. The scattering length, henceforth denoted by a , completely characterizes the interaction properties of atoms at low temperatures and low densities. The Feshbach resonance makes it possible to study one of the paradigms [12, 13, 14] of fermion pairing theory, the BCS-BEC crossover, experimentally.

Experimental groups have already observed the formation of thermal gases [15, 16] and Bose-Einstein condensates [17, 18, 19] of diatomic molecules. Condensation of fermionic atom pairs on the attractive interaction side of the resonance, where there is no two-atom bound state and the analogy to the BCS-BEC crossover problem is closer, has also been reported [20, 21]. Because the BCS transition does not manifest itself strongly [22] in the expanded density profile of the gas there is a need for quantitative and direct measurements of pairing correlations, one that has motivated a large number of proposals. Several works focused on the change of light scattering due to the transition from the normal to the superfluid phase as a detection method [23, 24, 25, 26]. Later, resonant laser light was proposed to induce tunnelling between the superfluid and normal state of the gas [27]. The experimental realization of Bragg spectroscopy in a Bose gas [28] has inspired theoretical work on pairing effects in the

dynamic structure function [29, 30]. Other interesting proposals include ones based on pairing induced changes in collective-mode frequencies [31, 32, 33, 34, 35, 36], rotational properties of the gas [37, 38], the expansion of the gas [39], and atomic density noise-correlation properties [40].

In this Letter we propose a more direct probe of pairing correlations that is similar to the spin magnetic susceptibility and nuclear-spin relaxation probes commonly used to detect electron pairing in condensed matter systems, and is able to detect pairing even when it does not lead to long-range coherence. We suggest a measurement of the cost in interaction energy when the number of Cooper pairs in the system is reduced by making the hyperfine state populations unequal. As we show below, this energy change can be extracted from data obtained using the RF spectroscopy techniques that have already been developed by several experimental groups [41, 42, 43]. To illustrate the direct relationship between the hyperfine population dependence of the interaction energy and pairing correlations, we compare the predictions of BCS theory for this quantity with its predictions for the more familiar magnetic susceptibility probe, which measures instead the dependence of total (interaction plus kinetic) energy on the same variable.

Interaction Energy Sum Rule — We consider a gas of fermionic atoms that consists of a mixture of two hyperfine states denoted by $|\uparrow\rangle$ and $|\downarrow\rangle$. We assume the temperatures to be low enough so that only s -channel interactions, forbidden between atoms in the same hyperfine state by the Pauli principle, are significant. The Hamiltonian of the gas is therefore $H = H_0 + H_{\text{int}}$, where the noninteracting part is

$$H_0 = \int d\mathbf{x} \sum_{\alpha=\{\uparrow,\downarrow\}} \psi_{\alpha}^{\dagger}(\mathbf{x}) \left(-\frac{\hbar^2 \nabla^2}{2m} + \epsilon_{\alpha} \right) \psi_{\alpha}(\mathbf{x}), \quad (1)$$

and ψ_{α} is the fermionic annihilation operator for hyperfine state $|\alpha\rangle$. The internal Zeeman energy of a hyperfine state is denoted by ϵ_{α} , and for simplicity we have neglected any inhomogeneity of the magnetic field. In particular, this implies that we neglect the effects of the

magnetic trapping potential. We take the interaction between unlike hyperfine states to be a contact interaction with strength $V_{\uparrow\downarrow}$, which should be chosen to produce the correct two-body scattering amplitude [44]. With these assumptions, the interaction part of the Hamiltonian is

$$H_{\text{int}} = V_{\uparrow\downarrow} \int d\mathbf{x} \psi_{\uparrow}^{\dagger}(\mathbf{x}) \psi_{\downarrow}^{\dagger}(\mathbf{x}) \psi_{\downarrow}(\mathbf{x}) \psi_{\uparrow}(\mathbf{x}) . \quad (2)$$

In the RF experiments one of the system hyperfine species (say $|\downarrow\rangle$) is coupled to a spectator hyperfine state ($|\text{s}\rangle$), and the number of atoms in the spectator state, N_s , is detected as a function of the frequency of the RF-field. In the linear response limit N_s is proportional to the rate of $|\downarrow\rangle \rightarrow |\text{s}\rangle$, transitions which we denote by $I(\omega)$. We define the position of the associated RF spectrum absorption line as

$$\hbar\bar{\omega} \equiv \frac{\int d\omega \hbar\omega I(\omega)}{\int d\omega I(\omega)} . \quad (3)$$

Using a formal golden-rule expression, $I(\omega)$ can be expressed in terms of a two-particle correlation function of Fermion fields. It then follows from the fermion analog of sum rules derived in Refs. [45, 46] that

$$\hbar\bar{\omega} = \hbar\omega_0 + \frac{1}{n_{\uparrow}} (V_{\uparrow\uparrow} - V_{\text{s}\uparrow}) \langle \psi_{\uparrow}^{\dagger} \psi_{\downarrow}^{\dagger} \psi_{\uparrow} \psi_{\downarrow} \rangle , \quad (4)$$

where $V_{\text{s}\uparrow}$ denotes the strength of the contact interaction between the spectator and $|\uparrow\rangle$ hyperfine states, and n_{α} denotes the average density in spin state $|\alpha\rangle$. The shift of $\hbar\bar{\omega}$ from the bare line position $\hbar\omega_0 = \epsilon_{\downarrow} - \epsilon_{\text{s}}$ differs from the interaction energy per volume by a factor, $V_{\uparrow\uparrow}n_{\uparrow}/(V_{\uparrow\uparrow} - V_{\text{s}\uparrow})$, which can be held constant through the experiments and if necessary can be accurately determined by separate measurements. We conclude that the RF spectra enable a direct measurement of the interaction energy density

$$e_{\text{int}}(n_{\uparrow}, n_{\downarrow}) \equiv V_{\uparrow\downarrow} \langle \psi_{\uparrow}^{\dagger} \psi_{\downarrow}^{\dagger} \psi_{\downarrow} \psi_{\uparrow} \rangle . \quad (5)$$

Pairing and Interaction Energy in BCS Theory — The inverse spin-susceptibility of an unpolarized system of spin-1/2 particles may be expressed in terms of the dependence of free energy on spin-polarization:

$$\chi_s^{-1} = \left. \frac{\partial^2 f_{\text{tot}}}{\partial \delta n^2} \right|_n = \frac{1}{2} \left[\frac{\partial^2 f_{\text{tot}}(n_{\uparrow}, n_{\downarrow})}{\partial n_{\uparrow}^2} - \frac{\partial^2 f_{\text{tot}}(n_{\uparrow}, n_{\downarrow})}{\partial n_{\uparrow} \partial n_{\downarrow}} \right] , \quad (6)$$

where f_{tot} is the total free energy per unit volume of the gas, n is the total density, and $\delta n \equiv n_{\uparrow} - n_{\downarrow}$ is the spin density. It is well-known that the χ_s is strongly suppressed when atoms can gain energy by pairing. (χ_s vanishes as $T \rightarrow 0$ in the BCS state.) For attractive atom-atom interactions, the energy cost of finite-spin polarization has positive contributions from both interaction and kinetic energy. In the following we compare the spin-susceptibility with an alternate quantity that is defined in terms of the interaction energy alone and can be extracted from RF spectroscopy experiments performed for a series of hyperfine-state populations:

$$\chi_{\text{int},s}^{-1} = \left. \frac{\partial^2 e_{\text{int}}(n_{\uparrow}, n_{\downarrow})}{\partial \delta n^2} \right|_n . \quad (7)$$

As we show below, this quantity and the inverse spin-susceptibility provide similar probes of pairing correlations.

We evaluate this *interaction susceptibility* using BCS theory from which it follows that

$$e_{\text{int}}(n_{\uparrow}, n_{\downarrow}) = \frac{|\Delta|^2}{V_{\uparrow\downarrow}} + \frac{4\pi a \hbar^2 n_{\uparrow} n_{\downarrow}}{m} , \quad (8)$$

where the dependence of the gap $\Delta \equiv V_{\uparrow\downarrow} \langle \psi_{\downarrow} \psi_{\uparrow} \rangle$ on temperature and hyperfine densities can be determined by solving the self-consistent mean-field equations. The mean-field Hamiltonian is [22]

$$\begin{aligned} H_0 = \int d\mathbf{x} \left\{ \psi_{\uparrow}^{\dagger}(\mathbf{x}) \left(-\frac{\hbar^2 \nabla^2}{2m} + \frac{4\pi a \hbar^2 n_{\downarrow}}{m} - \mu_{\uparrow} \right) \psi_{\uparrow}(\mathbf{x}) + \psi_{\downarrow}^{\dagger}(\mathbf{x}) \left(-\frac{\hbar^2 \nabla^2}{2m} + \frac{4\pi a \hbar^2 n_{\uparrow}}{m} - \mu_{\downarrow} \right) \psi_{\downarrow}(\mathbf{x}) \right. \\ \left. + \Delta \psi_{\uparrow}^{\dagger}(\mathbf{x}) \psi_{\downarrow}^{\dagger}(\mathbf{x}) + \Delta^* \psi_{\downarrow}(\mathbf{x}) \psi_{\uparrow}(\mathbf{x}) - \frac{|\Delta|^2}{V_{\uparrow\downarrow}} - \frac{4\pi a \hbar^2 n_{\uparrow} n_{\downarrow}}{m} \right\} , \end{aligned} \quad (9)$$

Note that the renormalization $V_{\uparrow\downarrow} \rightarrow 4\pi a \hbar^2/m$ can be made at this stage in the Hartree mean-field potential. The chemical potentials of the two hyperfine states are denoted by μ_{α} , and are not necessarily equal, thus allowing for a density difference between the two hyperfine

states.

The partial densities are given by

$$n_{\alpha} = \int \frac{d\mathbf{k}}{(2\pi)^3} \left\{ |u_{\mathbf{k}}|^2 N(\hbar\omega_{\mathbf{k},\alpha}) + |v_{\mathbf{k}}|^2 [1 - N(\hbar\omega_{\mathbf{k},-\alpha})] \right\} , \quad (10)$$

where $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ are the Bogoliubov coherence factors, $N(x) = [e^{\beta x} + 1]^{-1}$ is the Fermi distribution function, and the $|\uparrow\rangle$ quasiparticle dispersion is given by

$$\hbar\omega_{\mathbf{k},\uparrow} = \frac{\mu'_{\downarrow} - \mu'_{\uparrow}}{2} + \sqrt{[\epsilon_{\mathbf{k}} - (\mu'_{\uparrow} + \mu'_{\downarrow})/2]^2 + |\Delta|^2}. \quad (11)$$

An identical expression, with the hyperfine labels interchanged, applies for $\hbar\omega_{\mathbf{k},\downarrow}$. The Hartree-Fock mean-field shift is absorbed in the chemical potential via $\mu'_{\alpha} = \mu_{\alpha} - 4\pi a \hbar^2 n_{-\alpha}/m$. These equations for the densities need to be solved together with the BCS gap equation

$$\int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1 - N(\hbar\omega_{\mathbf{k},\uparrow}) - N(\hbar\omega_{\mathbf{k},\downarrow})}{2\sqrt{[\epsilon_{\mathbf{k}} - (\mu_{\uparrow} + \mu_{\downarrow})/2]^2 + |\Delta|^2}} = -\frac{1}{V_{\uparrow\downarrow}}. \quad (12)$$

Eq. (12) contains a ultraviolet divergence that is renormalized by using introducing the s -wave scattering length (a) between $|\uparrow\rangle$ and $|\downarrow\rangle$ hyperfine states by means of the Lippmann-Schwinger equation

$$\frac{m}{4\pi a \hbar^2} = \frac{1}{V_{\uparrow\downarrow}} + \int_{\mathbf{k} \leq k_{\Lambda}} \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{2\epsilon_{\mathbf{k}}}, \quad (13)$$

where we have introduced an ultraviolet cutoff k_{Λ} . It follows [22] from Eq. (13) that

$$V_{\uparrow\downarrow} = \frac{4\pi a \hbar^2}{m} \frac{1}{1 - \frac{2ak_{\Lambda}}{\pi}}. \quad (14)$$

For $k_{\Lambda} \simeq (100a_0)^{-1}$, and $a = -2000a_0$, we find that $V_{\uparrow\downarrow} \simeq 0.07 \times (4\pi a \hbar^2/m)$. Note that although the spin densities and the BCS gap parameter are independent of the short-range properties of the interatomic potential, the interaction energy in Eq. (8) is not [22].

Above the critical temperature, $T_{\text{BCS}} \simeq 0.6T_F e^{-\pi/2k_F|a|}$, $\Delta \rightarrow 0$ so that

$$e_{\text{int}}(n_{\uparrow}, n_{\downarrow}) = \frac{4\pi a \hbar^2 n_{\uparrow} n_{\downarrow}}{m}, \quad (15)$$

and $\chi_{\text{int},s}^{-1} = 2\pi|a|\hbar^2/m$ is temperature independent. For $T < T_{\text{BCS}}$ the interaction energy is given by Eq. (8). Linearization of the gap equation implies that for $T \uparrow T_{\text{BCS}}$,

$$\chi_{\text{int},s}^{-1} = \frac{\pi}{0.07k_F|a|} \left(\frac{\partial n}{\partial \mu} \right)^{-1} + 2\pi|a|\hbar^2/m. \quad (16)$$

In Fig. 1 we plot $\chi_{\text{int},s}^{-1}$ and the inverse susceptibility *vs.* temperature, for the case of $n_{\uparrow} = n_{\downarrow} = n/2$, scattering length $a = -2000a_0$ where a_0 is the Bohr radius, and density $n = 10^{12} \text{ cm}^{-3}$. For these parameters $T_{\text{BCS}} \simeq 0.005 \times T_F$. The inverse interaction susceptibility is greatly enhanced for temperatures below the BCS transition temperature, thus providing a clear signature of atomic Cooper pairs.

Discussion and Conclusions — Although we have so far assumed a homogeneous Fermi gas, we believe that

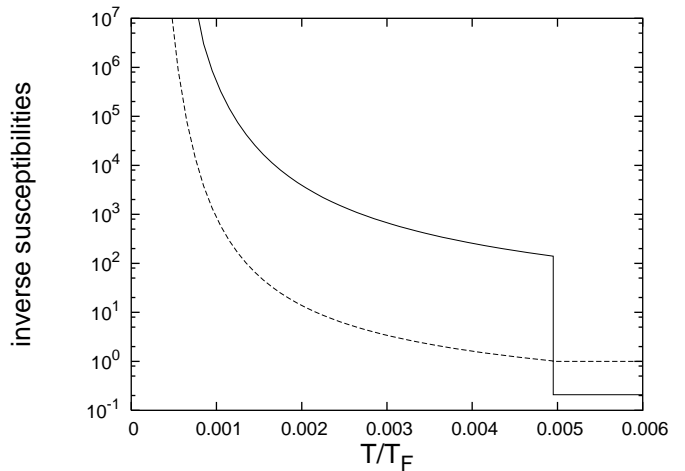


FIG. 1: Plot of $\chi_{\text{int},s}^{-1}$ (solid line) and χ_s^{-1} (dashed line) *vs.* temperature. Both susceptibilities are plotted in units of the quasiparticle density-of-states $(\partial n/\partial \mu)$. Note that the mean-field-theory interaction susceptibility is discontinuous at the critical temperature.

our results also apply to the case of a trapped, and therefore, inhomogeneous Fermi gas. There are two main reasons for this. First, since the inhomogeneity in the density profile leads to broadening of the RF spectrum, the sum rule in Eq. (3) is valid with the density n_{\uparrow} replaced by a mean density [42, 43]. Second, because the inverse Fermi wave vector is much smaller than the harmonic oscillator length for the experimental systems of interest, the system may be treated within a local density approximation [22]. Since the BCS gap parameter Δ will be largest in the center of the trap, the result in Eq. (16) should be evaluated at the center of the trap. With these two modifications, the results presented in this Letter should carry over, in a more than qualitative manner, to the inhomogeneous case.

For strong attractive interactions, $k_F|a| \sim 1$, mean-field theory is not expected to be accurate. In particular, the superfluid transition temperature is expected to be limited by the loss of long range coherence rather than by the thermodynamics of pair formation. The thermodynamic probe we discuss here is sensitive to the occurrence of pair correlations and not particularly sensitive to the establishment of long range coherence. It should therefore be able to detect the gradual development of pairing correlations with increasing interaction strength as the superfluid state is approached. We note that Bourdel *et al.* [47] have measured the ratio of the interaction energy and kinetic energy of a Fermi gas by comparing expanded density profiles of an interacting gas of atoms with expansion profiles of a gas at zero scattering length. In the weak-coupling limit such a measurement would provide direct information on the temperature dependence of the interaction energy, since the kinetic energy is almost independent of temperature in this case. In the

strong-coupling limit it is, however, not clear how the kinetic energy depends on the density difference, and it is, therefore, not obvious that a measurement of the ratio of interaction and kinetic energy for different hyperfine state populations would provide a sensitive probe of pairing correlations in the gas in this limit. Finally we remark on the possibility of realizing inhomogeneous pair-condensate states [48] in cold atom systems with unbalanced hyperfine state populations. These states could be detected by bringing the system to equilibrium in a rotating reference frame and visualizing their unusual [49] vortex-lattice structures.

This work was supported by the National Science Foundation under grant DMR-0115947 and by the Welch Foundation. We are grateful for informative conversations with D.S. Jin.

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